Sylver Coinage Positions with g=2

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# The king made silver as common as stones…

*1 Kings 10:27*

# 1 Introduction

Sylver Coinage positions can be exhaustively analysed when *g*=1, given a computer and enough time. However, when *g*=2, while the positions are still able to be analysed, it takes much more work.

It is easy to see that {2} is answered by 3, and with only slightly more work can find that {4} is answered by 6 and vice versa, and the same for all *g*=2 derived positions.

Thus, the first interesting case to examine is the winning moves in {8}. This has been completely analysed previously, however the winning moves have not been given, aside from a few cases with exceptionally high odd winning move.

Here full analysis is given of both the positions in {8}, and those in {10}. 𝓟-positions will be marked 𝓟. Positions not in canonical form (where some numbers are eliminated by others) are marked X.

While one winning move is given in each position, there may (and in many cases are) other winning moves. For exceptional values outside of the tables the open bracket notation is used, but for those in the tables just the winning move is written.

# 2 Eight

The simple pairing strategy (4*n*+1,4*n*+3) suffices for the 𝓟-positions {8,12} and {8,12,8*n*+2,8*n*+6}, as one is available whenever the other is.

The remainder of the positions will be organized into tables. Each table corresponds to which other multiple of 4 is in the position. Numbers of the form 8*n*+2 will be along the top, and those of the form 8*n*+6 will be along the sides:

## 2.1 {8,20}

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| {8,20} | {18} | {26} | {34} | {42} | {50} | {58} | {} |
| {14} | 9 | 9 | X | X | X | X | 9 |
| {22} | 11 | 13 | 13 | X | X | X | 13 |
| {30} | 11 | 12 | 19 | 23 | X | X | 𝓟 |
| {38} | X | 21 | 12 | 19 | 141 | X | 30 |
| {46} | X | X | 26 | 26 | 26 | 26 | 26 |
| {} | 11 | 𝓟 | 26 | 26 | 26 | 26 | 26 |

## 2.2 {8,28}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| {8,28} | {18} | {26} | {34} | {42} | {} |
| {22} | 12 | 91 | 15 | 14 | 14 |
| {30} | 9 | 9 | 9 | 14 | 14 |
| {38} | 27 | 205 | 12 | 14 | 14 |
| {46} | X | 20 | 39 | 14 | 14 |
| {54} | X | X | 31 | 14 | 14 |
| {62} | X | X | X | 14 | 14 |
| {} | 𝓟 | 18 | 18 | 14 | 14 |

## 2.3 {8,36}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {8,36} | {26} | {34} | {42} | {} |
| {22} | 25 | 39 | 14 | 14 |
| {30} | 12 | 25 | 14 | 14 |
| {38} | 27 | 12 | 14 | 14 |
| {46} | 20 | 71 | 14 | 14 |
| {54} | 20 | 173 | 14 | 14 |
| {62} | X | 647 | 14 | 14 |
| {70} | X | X | 14 | 14 |
| {} | 20 | 𝓟 | 14 | 14 |

## 2.4 {8,44}

|  |  |  |  |
| --- | --- | --- | --- |
| {8,44} | {26} | {34} | {} |
| {30} | 12 | 61 | 14 |
| {38} | 15 | 12 | 14 |
| {46} | 20 | 201 | 14 |
| {54} | 20 | 57 | 14 |
| {62} | 20 | 101 | 14 |
| {70} | X | 36 | 14 |
| {78} | X | X | 14 |
| {} | 20 | 36 | 14 |

## 2.5 {8,52}

|  |  |  |
| --- | --- | --- |
| {8,52} | {34} | {} |
| {30} | 393 | 14 |
| {38} | 12 | 14 |
| {46} | 135 | 14 |
| {54} | 115 | 14 |
| {62} | 37 | 14 |
| {70} | 36 | 14 |
| {} | 36 | 14 |

## 2.6 {8,60}

|  |  |  |
| --- | --- | --- |
| {8,60} | {34} | {} |
| {38} | 12 | 14 |
| {46} | 19 | 14 |
| {54} | 19 | 14 |
| {62} | 19 | 14 |
| {70} | 36 | 14 |
| {} | 36 | 14 |

## 2.7 Just {8}

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| {8} | {10} | {18} | {26} | {34} | {} |
| {14} | 12 | 25 | 17 | 27 | 𝓟 |
| {22} | 𝓟 | 12 | 10 | 10 | 14 |
| {30} | 22 | 13 | 12 | 49337 | 14 |
| {38} | 22 | 28 | 83 | 12 | 14 |
| {46} | 22 | 28 | 20 | 171 | 14 |
| {54} | 22 | 28 | 20 | 107 | 14 |
| {62} | 22 | 28 | 20 | 43 | 14 |
| {70} | 22 | 28 | 20 | 36 | 14 |
| {} | 22 | 28 | 20 | 36 | 14 |

# 3 Ten

The analysis of {10} is more complicated, so it will be broken down into sections based on the second smallest number in the position. Each section will have multiple tables, along with a few exceptional cases which if included in the tables would make them unwieldy. {10} is answered by 5,14, and 26. Therefore we can write off any position with all other values larger than *t̅*(10,14)=46.

## 3.1 {10,12} positions

{10,12} has a winning move of 7, so all positions without 16 or 18 also have 7 as a winning move. All positions with 14 have 8 as a winning move. The other positions are {10,12,16,18} [29, {10,12,16} [31, {10,12,18,26} [5, and {10,12,18} which is a 𝓟-position.

## 3.2 {10,14} positions

The positions containing 32, 36, or 46 are:

{10,14,46} [27, {10,14,32,36} [13, {10,14,36} [73, {10,14,26,32} [13, {10,14,32} [13

The remaining positions are:

|  |  |  |  |
| --- | --- | --- | --- |
| {10,14} | {16} | {26} | {} |
| {18,22} | 7 | 29 | 𝓟 |
| {18} | 𝓟 | 16 | 16 |
| {22} | 11 | 101 | 18 |
| {} | 18 | 293 | 𝓟 |

## 3.3 {10,16} positions

{10,16} is answered by 9, and so are all derived positions not containing 22 or 24. The positions containing 22 are answered by 8. The remaining positions are:

{10,16,18,24} [11, {10,16,24,28} [5, {10,16,24,38} [47, and {10,16,24}, which is a 𝓟-position.

## 3.4 {10,18} positions

All positions with 22 are answered by 8, and all positions without 26 are answered by 12. The remaining positions are:

|  |  |  |  |
| --- | --- | --- | --- |
| {10,18,26} | {32} | {42} | {} |
| {24} | 27 | X | 25 |
| {34} | 21 | 21 | 𝓟 |
| {} | 49 | 49 | 34 |

## 3.5 {10,22} positions

All {10,22} positions are answered by 8.

## 3.6 {10,24} positions

The position {10,24} is answered by 16, and so is every derived position not containing 28 or 38.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,24,28} | {26} | {36} | {46} | {} |
| {32} | 11 | 9 | 9 | 9 |
| {42} | 11 | 15 | 49 | 14 |
| {} | 11 | 129 | 25 | 14 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,24,38} | {26} | {36} | {46} | {56} | {66} | {} |
| {32} | 5 | 9 | 9 | X | X | 9 |
| {42} | 27 | 11 | 41 | 14 | 14 | 14 |
| {52} | X | 11 | 19 | 14 | 14 | 14 |
| {} | 117 | 11 | 277 | 14 | 14 | 14 |

## 3.7 {10,26} positions

This section is organized into three tables: The positions containing 32, those containing 42, and those containing neither.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,26,32} | {34} | {44} | {54} | {} |
| {28} | 19 | 25 | X | 217 |
| {38} | 29 | 11 | 11 | 11 |
| {48} | 15 | 19 | 61 | 41 |
| {} | 71 | 19 | 93 | 35 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,26,42} | {34} | {44} | {54} | {64} | {74} | {} |
| {28} | 15 | 35 | X | X | X | 49 |
| {38} | 65 | 11 | 11 | X | X | 11 |
| {48} | 75 | 73 | 19 | 13 | X | 4455 |
| {58} | 69 | 71 | 19 | 81 | 67 | 𝓟 |
| {} | 𝓟 | 34 | 34 | 34 | 34 | 34 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,26} | {34} | {44} | {54} | {64} | {74} | {84} | {94} | {} |
| {28} | 18 | 23 | X | X | X | X | X | 13 |
| {38} | 18 | 11 | 11 | X | X | X | X | 11 |
| {48} | 18 | 5 | 123 | 25 | X | X | X | 127 |
| {58} | 18 | 81 | 75 | 189 | 143 | X | X | 42 |
| {68} | X | 13 | 71 | 75 | 25 | 143 | X | 3899 |
| {} | 18 | 2383 | 177 | 45 | 121 | 81 | 251 | 𝓟 |

## 3.8 {10,28} positions

The position has 14 as a winning move, as do all derived positions not containing 32, 36, or 46.

|  |  |  |  |
| --- | --- | --- | --- |
| {10,28,32} | {36} | {46} | {} |
| {34} | 15 | 39 | 101 |
| {44} | 31 | 5 | 11 |
| {54} | 233 | 27 | 5 |
| {} | 𝓟 | 36 | 36 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,28,36} | {42} | {52} | {62} | {} |
| {34} | 51 | 109 | X | 77 |
| {44} | 77 | 15 | 19 | 𝓟 |
| {54} | 27 | 13 | 13 | 13 |
| {} | 32 | 32 | 32 | 32 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,28,46} | {42} | {52} | {62} | {72} | {82} | {} |
| {34} | 59 | 19 | X | X | X | 71 |
| {44} | 23 | 77 | 55 | X | X | 36 |
| {54} | 15 | 13 | 13 | 13 | X | 13 |
| {64} | 231 | 13 | 13 | 13 | 13 | 13 |
| {} | 3293 | 13 | 13 | 13 | 13 | 13 |

## 3.9 {10,32} positions

This section is divided into four tables, containing 34, 44, 54, and none of them respectively.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,32,34} | {38} | {48} | {58} | {} |
| {36} | 19 | 19 | 19 | 19 |
| {46} | 45 | 27 | 15 | 75 |
| {56} | 49 | 83 | 27 | 17 |
| {} | 2391 | 61 | 39 | 115 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,32,44} | {38} | {48} | {58} | {68} | {78} | {} |
| {36} | 15 | 5 | 47 | X | X | 1117 |
| {46} | 27 | 79 | 45 | 39 | X | 571 |
| {56} | 31 | 35 | 59 | 17 | 17 | 17 |
| {66} | 409 | 49 | 29 | 29 | 29 | 29 |
| {} | 𝓟 | 38 | 38 | 38 | 38 | 38 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,32,54} | {38} | {48} | {58} | {68} | {78} | {88} | {98} | {} |
| {36} | 41 | 45 | 39 | X | X | X | X | 53 |
| {46} | 99 | 101 | 25 | 57 | X | X | X | 15 |
| {56} | 19 | 19 | 19 | 17 | 17 | X | X | 17 |
| {66} | 85 | 89 | 15 | 419 | 25 | 37 | X | 499 |
| {76} | X | 65 | 53 | 15 | 27 | 45 | 197 | 𝓟 |
| {} | 44 | 4539 | 27 | 435 | 89 | 41 | 55 | 76 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,32} | {38} | {48} | {58} | {68} | {78} | {88} | {98} | {108} | {118} | {} |
| {36} | 28 | 28 | 28 | X | X | X | X | X | X | 28 |
| {46} | 19 | 19 | 19 | 19 | X | X | X | X | X | 19 |
| {56} | 71 | 15 | 425 | 17 | 17 | X | X | X | X | 17 |
| {66} | 5 | 79 | 21 | 25 | 45 | 43 | X | X | X | 5221 |
| {76} | X | 127 | 5 | 39 | 27 | 21 | 337 | X | X | 54 |
| {86} | X | 47 | 27 | 51 | 39 | 39 | 39 | 39 | X | 39 |
| {} | 44 | 𝓟 | 48 | 48 | 48 | 48 | 48 | 48 | 48 | 48 |

## 3.10 {10,34} positions

The position {10,34} has 14 as a winning move, as do all derived positions not containing 36 or 46.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,34,36} | {38} | {48} | {58} | {} |
| {42} | 21 | 53 | 29 | 29 |
| {52} | 21 | 13 | 41 | 129 |
| {62} | 21 | 13 | 31 | 141 |
| {} | 21 | 13 | 31 | 63 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| {10,34,46} | {38} | {48} | {58} | {} |
| {42} | 57 | 17 | 33 | 51 |
| {52} | 99 | 13 | 95 | 31 |
| {62} | 15 | 13 | 5 | 25 |
| {72} | X | 13 | 55 | 5 |
| {82} | X | X | 275 | 15 |
| {} | 79 | 13 | 𝓟 | 353 |

## 3.11 {10,36} positions

This section is divided into four tables, containing 42, 52, 62, and none of them respectively.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| {10,36,42} | {38} | {48} | {58} | {68} | {} |
| {44} | 19 | 19 | 19 | 19 | 19 |
| {54} | 17 | 71 | 51 | 39 | 77 |
| {64} | 17 | 13 | 127 | 5 | 57 |
| {74} | X | 83 | 55 | 33 | 21 |
| {} | 17 | 𝓟 | 48 | 48 | 48 |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,36,52} | {38} | {48} | {58} | {68} | {78} | {} |
| {44} | 57 | 89 | 191 | 13 | 44 | 𝓟 |
| {54} | 19 | 19 | 19 | 19 | 19 | 44 |
| {64} | 45 | 23 | 95 | 21 | 31 | 44 |
| {74} | X | 513 | 97 | 71 | 21 | 44 |
| {84} | X | X | 129 | 23 | 33 | 44 |
| {94} | X | X | X | 303 | 83 | 44 |
| {} | 75 | 42 | 8329 | 𝓟 | 26 | 26 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| {10,36,62} | {38} | {48} | {58} | {68} | {78} | {88} | {} |
| {44} | 29 | 33 | 43 | 13 | 31 | X | 52 |
| {54} | 101 | 5 | 81 | 89 | 127 | 209 | 9719 |
| {64} | 15 | 111 | 23 | 21 | 5 | 31 | 1445 |
| {74} | X | 69 | 15 | 5 | 21 | 21 | 21 |
| {84} | X | X | 29 | 29 | 29 | 29 | 29 |
| {94} | X | X | X | 151 | 33 | 47 | 45 |
| {104} | X | X | X | X | 26 | 26 | 26 |
| {114} | X | X | X | X | X | 26 | 26 |
| {} | 141 | 42 | 665 | 52 | 26 | 26 | 26 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,36} | {38} | {48} | {58} | {68} | {78} | {88} | {98} | {} |
| {44} | 28 | 28 | 28 | 28 | 28 | X | X | 28 |
| {54} | 37 | 35 | 29 | 29 | 29 | 29 | 29 | 29 |
| {64} | 51 | 5 | 89 | 21 | 189 | 31 | 455 | 5935 |
| {74} | X | 43 | 165 | 61 | 21 | 21 | 21 | 21 |
| {84} | X | X | 79 | 33 | 305 | 5 | 23 | 115 |
| {94} | X | X | X | 69 | 23 | 67 | 5 | 187 |
| {} | 15 | 42 | 93 | 839 | 26 | 26 | 26 | 26 |

## 3.12 {10,38}, {10,42}, and {10,44} positions

All of {10,38}, {10,42}, {10,44} has 14 as a winning move, as do all derived positions not containing 46.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,38,46} | {42} | {52} | {62} | {72} | {82} | {} |
| {44} | 55 | 5 | 17 | 25 | X | 177 |
| {54} | 5 | 15 | 17 | 47 | 145 | 𝓟 |
| {64} | 21 | 21 | 17 | 21 | 21 | 54 |
| {74} | 119 | 133 | 17 | 75 | 5 | 54 |
| {} | 𝓟 | 42 | 17 | 42 | 42 | 42 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| {10,42,46} | {48} | {58} | {68} | {78} | {} |
| {44} | 17 | 39 | 31 | 239 | 𝓟 |
| {54} | 17 | 91 | 73 | 81 | 44 |
| {64} | 17 | 29 | 29 | 5 | 29 |
| {74} | 17 | 45 | 35 | 63 | 15 |
| {} | 17 | 97 | 25 | 41 | 38 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| {10,44,46} | {48} | {58} | {68} | {78} | {} |
| {52} | 21 | 55 | 13 | 75 | 125 |
| {62} | 21 | 109 | 13 | 69 | 65 |
| {72} | 21 | 73 | 13 | 83 | 161 |
| {82} | 21 | 83 | 13 | 5 | 43 |
| {} | 21 | 201 | 13 | 117 | 5 |

## 3.13 {10,46} positions

This section is divided into five tables, containing 52, 62, 72, 82 and none of them respectively.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| {10,46,52} | {48} | {58} | {68} | {78} | {88} | {} |
| {54} | 31 | 111 | 5 | 95 | 131 | 6157 |
| {64} | 41 | 23 | 99 | 39 | 39 | 39 |
| {74} | 23 | 37 | 21 | 35 | 5 | 497 |
| {84} | 123 | 37 | 45 | 91 | 23 | 79 |
| {94} | X | 37 | 309 | 123 | 61 | 5 |
| {} | 2213 | 37 | 1759 | 26 | 26 | 26 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| {10,46,62} | {48} | {58} | {68} | {78} | {88} | {98} | {} |
| {54} | 19 | 19 | 19 | 19 | 19 | 19 | 19 |
| {64} | 77 | 67 | 39 | 55 | 43 | 221 | 853 |
| {74} | 5 | 47 | 21 | 153 | 165 | 89 | 2371 |
| {84} | 113 | 39 | 81 | 111 | 5 | 43 | 87 |
| {94} | X | 45 | 155 | 39 | 39 | 39 | 39 |
| {104} | X | X | 67 | 26 | 26 | 26 | 26 |
| {} | 101 | 25 | 42385 | 26 | 26 | 26 | 26 |

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,46,72} | {48} | {58} | {68} | {78} | {88} | {98} | {108} | {} |
| {54} | 103 | 41 | 63 | 23 | 69 | 117 | X | 4865 |
| {64} | 23 | 5 | 55 | 59 | 147 | 35 | 69 | 71485 |
| {74} | 39 | 15 | 21 | 123 | 49 | 47 | 377 | 743 |
| {84} | 5 | 141 | 35 | 55 | 135 | 65 | 67 | 353 |
| {94} | X | 39 | 131 | 85 | 51 | 105 | 81 | 75 |
| {104} | X | X | 51 | 26 | 26 | 26 | 26 | 26 |
| {} | 135 | 185 | 2159 | 26 | 26 | 26 | 26 | 26 |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,46,82} | {48} | {58} | {68} | {78} | {88} | {98} | {108} | {118} | {} |
| {54} | 117 | 23 | 141 | 47 | 33 | 87 | X | X | 345 |
| {64} | 15 | 69 | 107 | 35 | 95 | 5 | 75 | 733 | 𝓟 |
| {74} | 61 | 63 | 21 | 393 | 107 | 73 | 105 | 165 | 64 |
| {84} | 47 | 15 | 65 | 61 | 83 | 289 | 199 | 809 | 64 |
| {94} | X | 61 | 5 | 69 | 41 | 109 | 187 | 193 | 64 |
| {104} | X | X | 47 | 26 | 26 | 26 | 26 | 26 | 26 |
| {} | 107 | 237 | 47 | 26 | 26 | 26 | 26 | 26 | 26 |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| {10,46} | {48} | {58} | {68} | {78} | {88} | {98} | {108} | {118} | {128} | {} |
| {54} | 193 | 49 | 25 | 39 | 39 | 39 | X | X | X | 23 |
| {64} | 185 | 63 | 113 | 61 | 35 | 57 | 59 | 15 | X | 82 |
| {74} | 47 | 137 | 21 | 25 | 81 | 35 | 197 | 51 | 57 | 27977 |
| {84} | 91 | 51 | 61 | 69 | 65 | 15 | 387 | 127 | 271 | 3553 |
| {94} | X | 67 | 107 | 63 | 25 | 137 | 15 | 121 | 47 | 1017 |
| {104} | X | X | 197 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |
| {} | 23 | 15 | 129 | 26 | 26 | 26 | 26 | 26 | 26 | 26 |

This concludes the analysis of 10, as all even moves higher than 46 can be answered with 14.

# 4 Conclusion and further questions:

## 4.1 Analysis of higher values

The natural next question is to ask for the winning moves to higher even positions. The next highest value to consider is 12. However, the only winning move known for 12 is 8, which, due to *g*({8,12}) being 4 still leaves infinitely many cases to consider. One even runs into trouble attempting to fully analyse {12,16}, and the completion requires knowing a good, odd reply to {12,16,20}, which so far, I do not know.

14 seems much more promising, as the winning move 8 restricts all non-trivial positions to have second lowest value at most 34. This may prove more difficult however, as the increased branching potential creates far more positions to analyse. I hope to eventually settle this case in its entirety; however, this may take considerably longer.

## 4.2 Nearly short is still quite long?

Define a nearly quiet ender *Q* to be an ender whose top move *t* is eliminated quietly by all but *t*/2, and a nearly short position *S* to be *Q*×*g*. These positions seem to have very large winning moves compared to other positions. Is there a reason behind this? Is it because it is almost but not quite possible to apply the Quiet End Theorem to prove that it is 𝓟, and the failure only catches up to the position at very high odd winning moves?