Sylver Coinage Positions with g=2

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The king made silver as common as stones…

1 Kings 10:27

1 Introduction

Sylver Coinage positions can be exhaustively analysed when g=1, given a computer and enough time. However, when g=2, while the positions are still able to be analysed, it takes much more work.

It is easy to see that {2} is answered by 3, and with only slightly more work can find that {4} is answered by 6 and vice versa, and the same for all g=2 derived positions.

Thus, the first interesting case to examine is the winning moves in {8}. This has been completely analysed previously, however the winning moves have not been given, aside from a few cases with exceptionally high odd winning move.

Here full analysis is given of both the positions in {8}, and those in {10}. $\mathcal{P}$-positions will be marked $\mathcal{P}$. Positions not in canonical form (where some numbers are eliminated by others) are marked X.

While one winning move is given in each position, there may (and in many cases are) other winning moves. For exceptional values outside of the tables the open bracket notation is used, but for those in the tables just the winning move is written.

2 Eight

The simple pairing strategy ($4n$+1,$4n$+3) suffices for the $\mathcal{P}$-positions {8,12} and {8,12,$8n$+2,$8n$+6}, as one is available whenever the other is.

The remainder of the positions will be organized into tables. Each table corresponds to which other multiple of 4 is in the position. Numbers of the form $8n$+2 will be along the top, and those of the form $8n$+6 will be along the sides:

2.1 {8,20}

{8,20}	{18}	{26}	{34}	{42}	{50}	{58}	{}
{14}	9	9	X	X	X	X	9
{22}	11	13	13	X	X	X	13
{30}	11	12	19	23	X	X	$\mathcal{P}$
{38}	X	21	12	19	141	X	30
{46}	X	X	26	26	26	26	26
{}	11	$\mathcal{P}$	26	26	26	26	26

2.2 {8,28}

{8,28}	{18}	{26}	{34}	{42}	{}
{22}	12	91	15	14	14
{30}	9	9	9	14	14
{38}	27	205	12	14	14
{46}	X	20	39	14	14
{54}	X	X	31	14	14
{62}	X	X	X	14	14
{}	$\mathcal{P}$	18	18	14	14

2.3 {8,36}

{8,36}	{26}	{34}	{42}	{}
{22}	25	39	14	14
{30}	12	25	14	14
{38}	27	12	14	14
{46}	20	71	14	14
{54}	20	173	14	14
{62}	X	647	14	14
{70}	X	X	14	14
{}	20	$\mathcal{P}$	14	14

2.4 {8,44}

{8,44}	{26}	{34}	{}
{30}	12	61	14
{38}	15	12	14
{46}	20	201	14
{54}	20	57	14
{62}	20	101	14
{70}	X	36	14
{78}	X	X	14
{}	20	36	14

2.5 {8,52}

{8,52}	{34}	{}
{30}	393	14
{38}	12	14
{46}	135	14
{54}	115	14
{62}	37	14
{70}	36	14
{}	36	14

2.6 {8,60}

{8,60}	{34}	{}
{38}	12	14
{46}	19	14
{54}	19	14
{62}	19	14
{70}	36	14
{}	36	14

2.7 Just {8}

{8}	{10}	{18}	{26}	{34}	{}
{14}	12	25	17	27	$\mathcal{P}$
{22}	$\mathcal{P}$	12	10	10	14
{30}	22	13	12	49337	14
{38}	22	28	83	12	14
{46}	22	28	20	171	14
{54}	22	28	20	107	14
{62}	22	28	20	43	14
{70}	22	28	20	36	14
{}	22	28	20	36	14

3 Ten

The analysis of {10} is more complicated, so it will be broken down into sections based on the second smallest number in the position. Each section will have multiple tables, along with a few exceptional cases which if included in the tables would make them unwieldy. {10} is answered by 5,14, and 26. Therefore we can write off any position with all other values larger than $\bar{t}$(10,14)=46.

3.1 {10,12} positions

{10,12} has a winning move of 7, so all positions without 16 or 18 also have 7 as a winning move. All positions with 14 have 8 as a winning move. The other positions are {10,12,16,18} [29, {10,12,16} [31, {10,12,18,26} [5, and {10,12,18} which is a $\mathcal{P}$-position.

3.2 {10,14} positions

The positions containing 32, 36, or 46 are:
{10,14,46} [27, {10,14,32,36} [13, {10,14,36} [73, {10,14,26,32} [13, {10,14,32} [13

The remaining positions are:

{10,14}	{16}	{26}	{}
{18,22}	7	29	$\mathcal{P}$
{18}	$\mathcal{P}$	16	16
{22}	11	101	18
{}	18	293	$\mathcal{P}$

3.3 {10,16} positions

{10,16} is answered by 9, and so are all derived positions not containing 22 or 24. The positions containing 22 are answered by 8. The remaining positions are:
{10,16,18,24} [11, {10,16,24,28} [5, {10,16,24,38} [47, and {10,16,24}, which is a $\mathcal{P}$-position.

3.4 {10,18} positions

All positions with 22 are answered by 8, and all positions without 26 are answered by 12. The remaining positions are:

{10,18,26}	{32}	{42}	{}
{24}	27	X	25
{34}	21	21	$\mathcal{P}$
{}	49	49	34

3.5 {10,22} positions

All {10,22} positions are answered by 8.

3.6 {10,24} positions

The position {10,24} is answered by 16, and so is every derived position not containing 28 or 38.

{10,24,28}	{26}	{36}	{46}	{}
{32}	11	9	9	9
{42}	11	15	49	14
{}	11	129	25	14

{10,24,38}	{26}	{36}	{46}	{56}	{66}	{}
{32}	5	9	9	X	X	9
{42}	27	11	41	14	14	14
{52}	X	11	19	14	14	14
{}	117	11	277	14	14	14

3.7 {10,26} positions

This section is organized into three tables: The positions containing 32, those containing 42, and those containing neither.

{10,26,32}	{34}	{44}	{54}	{}
{28}	19	25	X	217
{38}	29	11	11	11
{48}	15	19	61	41
{}	71	19	93	35

{10,26,42}	{34}	{44}	{54}	{64}	{74}	{}
{28}	15	35	X	X	X	49
{38}	65	11	11	X	X	11
{48}	75	73	19	13	X	4455
{58}	69	71	19	81	67	$\mathcal{P}$
{}	$\mathcal{P}$	34	34	34	34	34

{10,26}	{34}	{44}	{54}	{64}	{74}	{84}	{94}	{}
{28}	18	23	X	X	X	X	X	13
{38}	18	11	11	X	X	X	X	11
{48}	18	5	123	25	X	X	X	127
{58}	18	81	75	189	143	X	X	42
{68}	X	13	71	75	25	143	X	3899
{}	18	2383	177	45	121	81	251	$\mathcal{P}$

3.8 {10,28} positions

The position has 14 as a winning move, as do all derived positions not containing 32, 36, or 46.

{10,28,32}	{36}	{46}	{}
{34}	15	39	101
{44}	31	5	11
{54}	233	27	5
{}	$\mathcal{P}$	36	36

{10,28,36}	{42}	{52}	{62}	{}
{34}	51	109	X	77
{44}	77	15	19	$\mathcal{P}$
{54}	27	13	13	13
{}	32	32	32	32

{10,28,46}	{42}	{52}	{62}	{72}	{82}	{}
{34}	59	19	X	X	X	71
{44}	23	77	55	X	X	36
{54}	15	13	13	13	X	13
{64}	231	13	13	13	13	13
{}	3293	13	13	13	13	13

3.9 {10,32} positions

This section is divided into four tables, containing 34, 44, 54, and none of them respectively.

{10,32,34}	{38}	{48}	{58}	{}
{36}	19	19	19	19
{46}	45	27	15	75
{56}	49	83	27	17
{}	2391	61	39	115

{10,32,44}	{38}	{48}	{58}	{68}	{78}	{}
{36}	15	5	47	X	X	1117
{46}	27	79	45	39	X	571
{56}	31	35	59	17	17	17
{66}	409	49	29	29	29	29
{}	$\mathcal{P}$	38	38	38	38	38

{10,32,54}	{38}	{48}	{58}	{68}	{78}	{88}	{98}	{}
{36}	41	45	39	X	X	X	X	53
{46}	99	101	25	57	X	X	X	15
{56}	19	19	19	17	17	X	X	17
{66}	85	89	15	419	25	37	X	499
{76}	X	65	53	15	27	45	197	$\mathcal{P}$
{}	44	4539	27	435	89	41	55	76

{10,32}	{38}	{48}	{58}	{68}	{78}	{88}	{98}	{108}	{118}	{}
{36}	28	28	28	X	X	X	X	X	X	28
{46}	19	19	19	19	X	X	X	X	X	19
{56}	71	15	425	17	17	X	X	X	X	17
{66}	5	79	21	25	45	43	X	X	X	5221
{76}	X	127	5	39	27	21	337	X	X	54
{86}	X	47	27	51	39	39	39	39	X	39
{}	44	$\mathcal{P}$	48	48	48	48	48	48	48	48

3.10 {10,34} positions

The position {10,34} has 14 as a winning move, as do all derived positions not containing 36 or 46.

{10,34,36}	{38}	{48}	{58}	{}
{42}	21	53	29	29
{52}	21	13	41	129
{62}	21	13	31	141
{}	21	13	31	63

{10,34,46}	{38}	{48}	{58}	{}
{42}	57	17	33	51
{52}	99	13	95	31
{62}	15	13	5	25
{72}	X	13	55	5
{82}	X	X	275	15
{}	79	13	$\mathcal{P}$	353

3.11 {10,36} positions

This section is divided into four tables, containing 42, 52, 62, and none of them respectively.

{10,36,42}	{38}	{48}	{58}	{68}	{}
{44}	19	19	19	19	19
{54}	17	71	51	39	77
{64}	17	13	127	5	57
{74}	X	83	55	33	21
{}	17	$\mathcal{P}$	48	48	48

{10,36,52}	{38}	{48}	{58}	{68}	{78}	{}
{44}	57	89	191	13	44	$\mathcal{P}$
{54}	19	19	19	19	19	44
{64}	45	23	95	21	31	44
{74}	X	513	97	71	21	44
{84}	X	X	129	23	33	44
{94}	X	X	X	303	83	44
{}	75	42	8329	$\mathcal{P}$	26	26

{10,36,62}	{38}	{48}	{58}	{68}	{78}	{88}	{}
{44}	29	33	43	13	31	X	52
{54}	101	5	81	89	127	209	9719
{64}	15	111	23	21	5	31	1445
{74}	X	69	15	5	21	21	21
{84}	X	X	29	29	29	29	29
{94}	X	X	X	151	33	47	45
{104}	X	X	X	X	26	26	26
{114}	X	X	X	X	X	26	26
{}	141	42	665	52	26	26	26

{10,36}	{38}	{48}	{58}	{68}	{78}	{88}	{98}	{}
{44}	28	28	28	28	28	X	X	28
{54}	37	35	29	29	29	29	29	29
{64}	51	5	89	21	189	31	455	5935
{74}	X	43	165	61	21	21	21	21
{84}	X	X	79	33	305	5	23	115
{94}	X	X	X	69	23	67	5	187
{}	15	42	93	839	26	26	26	26

3.12 {10,38}, {10,42}, and {10,44} positions

All of {10,38}, {10,42}, {10,44} has 14 as a winning move, as do all derived positions not containing 46.

{10,38,46}	{42}	{52}	{62}	{72}	{82}	{}
{44}	55	5	17	25	X	177
{54}	5	15	17	47	145	$\mathcal{P}$
{64}	21	21	17	21	21	54
{74}	119	133	17	75	5	54
{}	$\mathcal{P}$	42	17	42	42	42

{10,42,46}	{48}	{58}	{68}	{78}	{}
{44}	17	39	31	239	$\mathcal{P}$
{54}	17	91	73	81	44
{64}	17	29	29	5	29
{74}	17	45	35	63	15
{}	17	97	25	41	38

{10,44,46}	{48}	{58}	{68}	{78}	{}
{52}	21	55	13	75	125
{62}	21	109	13	69	65
{72}	21	73	13	83	161
{82}	21	83	13	5	43
{}	21	201	13	117	5

3.13 {10,46} positions

This section is divided into five tables, containing 52, 62, 72, 82 and none of them respectively.

{10,46,52}	{48}	{58}	{68}	{78}	{88}	{}
{54}	31	111	5	95	131	6157
{64}	41	23	99	39	39	39
{74}	23	37	21	35	5	497
{84}	123	37	45	91	23	79
{94}	X	37	309	123	61	5
{}	2213	37	1759	26	26	26

{10,46,62}	{48}	{58}	{68}	{78}	{88}	{98}	{}
{54}	19	19	19	19	19	19	19
{64}	77	67	39	55	43	221	853
{74}	5	47	21	153	165	89	2371
{84}	113	39	81	111	5	43	87
{94}	X	45	155	39	39	39	39
{104}	X	X	67	26	26	26	26
{}	101	25	42385	26	26	26	26

{10,46,72}	{48}	{58}	{68}	{78}	{88}	{98}	{108}	{}
{54}	103	41	63	23	69	117	X	4865
{64}	23	5	55	59	147	35	69	71485
{74}	39	15	21	123	49	47	377	743
{84}	5	141	35	55	135	65	67	353
{94}	X	39	131	85	51	105	81	75
{104}	X	X	51	26	26	26	26	26
{}	135	185	2159	26	26	26	26	26

{10,46,82}	{48}	{58}	{68}	{78}	{88}	{98}	{108}	{118}	{}
{54}	117	23	141	47	33	87	X	X	345
{64}	15	69	107	35	95	5	75	733	***P***
{74}	61	63	21	393	107	73	105	165	64
{84}	47	15	65	61	83	289	199	809	64
{94}	X	61	5	69	41	109	187	193	64
{104}	X	X	47	26	26	26	26	26	26
{}	107	237	47	26	26	26	26	26	26

{10,46}	{48}	{58}	{68}	{78}	{88}	{98}	{108}	{118}	{128}	{}
{54}	193	49	25	39	39	39	X	X	X	23
{64}	185	63	113	61	35	57	59	15	X	82
{74}	47	137	21	25	81	35	197	51	57	27977
{84}	91	51	61	69	65	15	387	127	271	3553
{94}	X	67	107	63	25	137	15	121	47	1017
{104}	X	X	197	26	26	26	26	26	26	26
{}	23	15	129	26	26	26	26	26	26	26

This concludes the analysis of 10, as all even moves higher than 46 can be answered with 14.

4 Conclusion and further questions:

4.1 Analysis of higher values

The natural next question is to ask for the winning moves to higher even positions. The next highest value to consider is 12. However, the only winning move known for 12 is 8, which, due to $g(\{8,12\})$ being 4 still leaves infinitely many cases to consider. One even runs into trouble attempting to fully analyse {12,16}, and the completion requires knowing a good, odd reply to {12,16,20}, which so far, I do not know.

14 seems much more promising, as the winning move 8 restricts all non-trivial positions to have second lowest value at most 34. This may prove more difficult however, as the increased branching potential creates far more positions to analyse. I hope to eventually settle this case in its entirety; however, this may take considerably longer.

4.2 Nearly short is still quite long?

Define a nearly quiet ender Q to be an ender whose top move t is eliminated quietly by all but $t/2$, and a nearly short position S to be $Q \times g$. These positions seem to have very large winning moves compared to other positions. Is there a reason behind this? Is it because it is almost but not quite possible to apply the Quiet End Theorem to prove that it is $\mathcal{P}$, and the failure only catches up to the position at very high odd winning moves?